

# A SIMPLE METHOD OF GOAL – DIRECTED LOSSY SYNTHESIS AND NETWORK OPTIMIZATION

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**Summary** A simple method for synthesis of goal – directed lossy filters is described in the present paper. This method of synthesis is based on the application of optimization procedures used in usually accessible professional software for network analysis. The presented method of network optimization enable to improve network performance without requirement of special numerical programs what brings new possibility for many designers in area of network design and optimization.

## 1. INTRODUCTION

By classical filter design there are usually single or double – sided termination RLC filter prototypes used. In many catalogues are these standard LC ladder prototypes which are using ideal loss – les reactive L and C elements wide tabled. Here the terminal resistances are transformed to the internal structure and dumped the LC circuit to realize the required transfer function.

In [2] was described a new method how to optimize resulting active filter structures based on classical ladder prototypes using goal – lossy filter prototype design, where the losses are dispersed to the whole ladder structure. It was shown that the performances of these filters designed with the lossy structures - like sensitivities, components ratio etc, may be better than the synthesis based on the classical loss – les RLC ladder prototypes.

Theoretically can be used goal lossy ladder prototypes with losses dispersed to the all branches of network. In the practice there are most often used structures, where losses are dispersed to all parallel (see Fig.1b) or to all serial branches of filter structure (Fig.1c) what corresponds with resulting parameters of active elements simulating passive ladder elements (inductors, FDNR) . In [2],[3], the design algorithm of these goal – lossy ladder

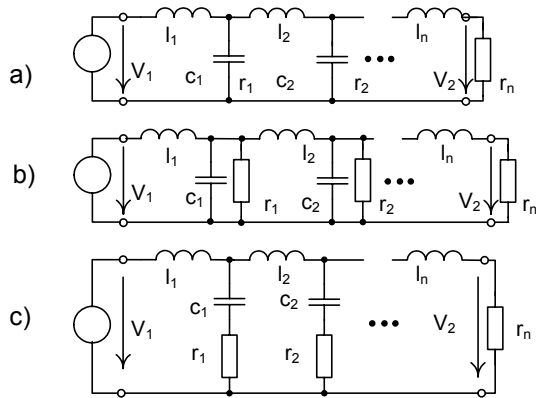


Fig. 1. RLC ladder filter prototype a) ideal, b) with parallel loses c) with serial loses

prototypes with parallel lossy resistors (Fig.1b) was described and special software to goal – lossy filter synthesis was presented, main possible network goal – lossy filter value elements was presented for standard types of approximations in form of tables too. However for many cases the width of approximation types or parameter ranges is not sufficient , requirement of special software is also some disadvantage for many designers. Therefore a new method how to design the goal – lossy filter structures has been developed.

## 2. A PRINCIPLE OF SYNTHESIS METHOD

In the present time there are at disposal many analysis software products [4],[5] which can be used generally by a process of network optimization. These products can be used also directly to synthesis of goal – directed lossy structures without requirement of the special numerical algorithm implementation or software. The software blocks called usually network optimizers and allow to

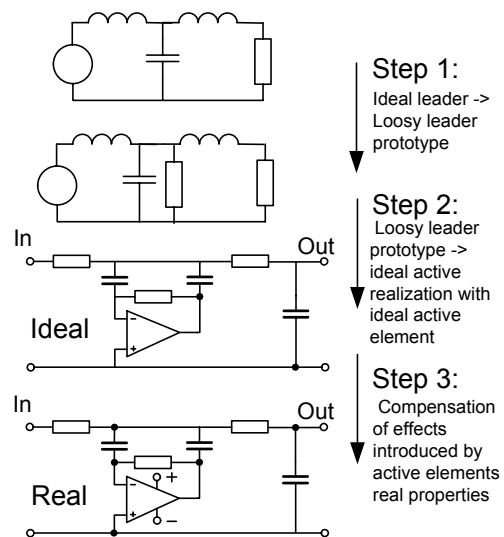


Fig. 2. The process of active filter design and process of final filter optimization

compute (in many cases by selected numerical method) all values of network elements with aims to reach the prescribed (ideal) transfer network characteristic. A principle and process of a new synthesis method is briefly given as rough draft in Fig.2.

The main procedures of filter component computing can be divided to these main steps:

- Lossy network analysis – analysis of lossy structure behavior and investigation of maximum lossy values or determination of limiting lossy parameters
- Generation of required transfer function table (transfer ratio value versus frequency),
- Final optimization of passive ladder network
- Final optimization of active network with regard to real active element performance.

### 3. THE SYNTHESIS OF GOAL - DIRECTED LOSSY PROTOTYPES WITH PARALLEL RESISTORS

The procedure of filter synthesis can be leaden to compute lossy ladder structure: a) to obtain the required quality factor of passive elements, b) to reach the transfer function with required resistor value. While in the above mentioned literature the first process was chosen, in presented new method the second possibility was selected.

The accuracy of goal lossy ladder prototype transfer response with comparison to transfer response of ideal loss-less ladder can be in a case of 3<sup>rd</sup> order ladder RLC filter analysis demonstrated. Transfer function of LP filters from Fig.1a) can be written as:

$$H(s) = \frac{1}{a_0 + a_1s^1 + a_2s^2 + \dots + a_n s^n}, \quad (1)$$

where  $a_0$ - $a_n$  are coefficients of denominator and  $s$  the complex frequency  $j\omega$ . The voltage transfer of goal - lossy structure from Fig.1b) is given:

$$H(s) = \frac{1}{a_0 + (a_1 + k_1)s^1 + (a_2 + k_2)s^2 + \dots + a_n s^n}, \quad (2)$$

where  $k_{1,2}$  are the parts which are inserted due to losses. By comparison of this two equation, we can observe, that the exact solution of problem (transfer function of goal lossy prototypes with parallel dumped resistors which is equivalent with ideal transfer response) evidently can be (in limited range of parameters ) found.

Applying above mentioned method by usage of software TINA optimizer [5] were calculated parameter values of two most often used standard

approximation filter types - Butterworth and Tscheybshev. The values (normalized to frequency  $\omega=1$ ) of parallel goal – lossy ladder prototypes for 5<sup>th</sup> to 9<sup>th</sup> filter order were ordered in table 1. The presented values can be directly to filter design of low – pass filters with FDNR active elements (as filter from Fig.3) or high – pass filters with active lossy (non – ideal) inductors successfully used.

As an example of low – pass filter design which has been based on derived lossy ladder prototype elements (From Table 1) here is presented resulting network of filter in Fig.3. In the presented low – pass filter are used as active elements simple parallel lossy FDNR networks with minimized numbers of operational amplifiers [1], [2].

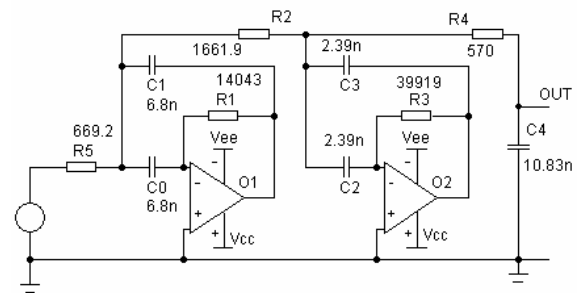


Fig. 3. The low – pass filter network designed using goal – directed lossy prototype

In the Fig.4 we can see a magnitude transfer response of designed (non optimized) active network with significant deviation from ideal transfer response due to real active element properties. The resulting transfer response (labeled as optimized) has been obtained after final filter optimization process (Step 3 of filter design and optimization process – in Fig.2), where the real active elements performance was eliminated.

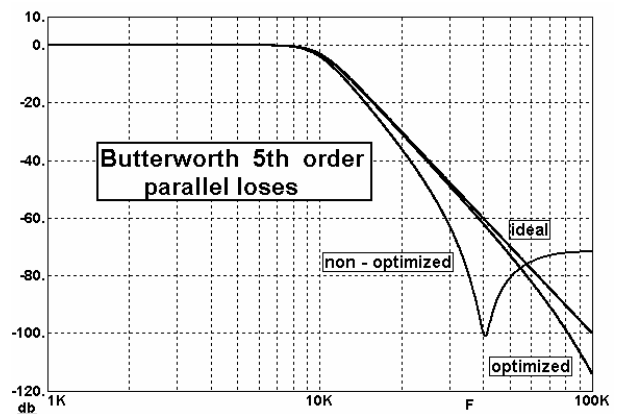


Fig. 4. The resulting transfer response of designed filter of 5<sup>th</sup> order low – pass Butterworth

#### 4. THE SYNTHESIS OF LOSSY GOAL – DIRECTED PROTOTYPES WITH SERIAL RESISTORS

The synthesis of goal – lossy network with serial resistors (Fig.1b) brings more problems. Transfer function of 3<sup>rd</sup> order mentioned filter network with serial lossy resistors can be expressed as:

$$H(s) = \frac{r_n + r_1 r_n c_1 s}{r_n + (r_1 r_n c_1 + l_1 + l_2)s + (r_1 l_1 + r_1 l_2 + r_n l_1) c_1 s^2 + l_2 c_1 l_1 s^3} \quad (3)$$

However by comparison with eq.(1), it is clear, that exact solution (equivalent transfer function by lossy structure) cannot be found. The nominator of transfer function (3) of goal - lossy RLC ladder prototype with serial dumped resistors exhibit inserted zeros of transfer with time constant  $r_n c_n$ . It means that from frequency  $\omega = 1/r_n c_n$  is the slope of transfer function reduced to -20dB . Therefore for this network structures the range of possible filter solutions is limited .

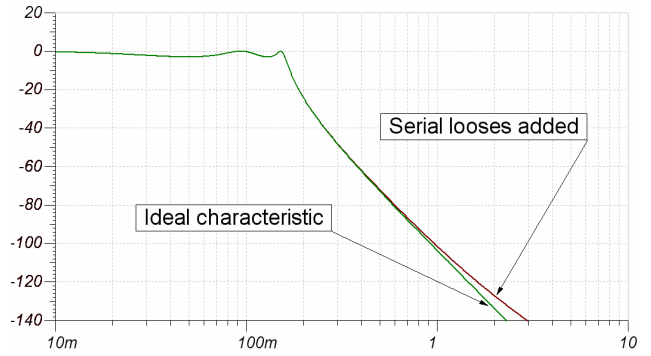


Fig. 5. The resulting transfer response in a case of network with serial lossy resistor prototype

By proper optimized network value parameters can be successfully found solution with allowable difference of transfer function from ideal required transfer response, how declare a typical example of transfer function of Tschebyshev goal – lossy prototype of 5<sup>th</sup> filter order from Fig.5. From comparison of transfer responses is evidently seen influence of parasitic pole for RLC lossy structure

Type	o	R <sub>p</sub>	l <sub>1</sub>	c <sub>1</sub>	l <sub>2</sub>	c <sub>2</sub>	l <sub>3</sub>	c <sub>3</sub>	l <sub>4</sub>	c <sub>4</sub>	l <sub>5</sub>
B U T T E R W O R T H	5	-	<b>1.5400</b>	<b>1.6900</b>	<b>1.3800</b>	<b>0.8940</b>	<b>0.3089</b>	-	-	-	-
		6	1.1040	2.1440	1.2400	1.0290	0.3336	-	-	-	-
		4	0.9630	2.3700	1.1590	1.0980	0.3450	-	-	-	-
		2	0.6928	3.0570	0.9805	1.2820	0.3726	-	-	-	-
		1	0.4370	4.4430	0.7501	1.6500	0.4112	-	-	-	-
	7	-	<b>1.5900</b>	<b>1.800</b>	<b>1.6600</b>	<b>1.4000</b>	<b>1.0500</b>	<b>0.6550</b>	<b>0.2220</b>	-	-
		6	0.9751	2.5700	1.2700	1.1800	0.9382	0.7628	0.2422	-	-
		4	0.8167	2.9500	1.1400	2.0100	0.8876	0.8132	0.2507	-	-
		2	0.5438	4.1200	0.8605	2.6100	0.7593	0.9585	0.2725	-	-
		1.5	0.4428	4.9000	0.7406	3.0000	0.6905	1.0500	0.2588	-	-
	9	-	<b>1.5600</b>	<b>1.8400</b>	<b>1.7800</b>	<b>1.6200</b>	<b>1.400</b>	<b>1.1400</b>	<b>0.8410</b>	<b>0.5150</b>	<b>0.1730</b>
		6	0.8628	2.9200	1.2000	2.3500	1.0800	1.4800	0.7567	0.6009	0.1869
T S C H E B Y S H E V	5	-	<b>2.1500</b>	<b>1.3000</b>	<b>2.6200</b>	<b>1.2500</b>	<b>1.7400</b>	-	-	-	-
		6	0.8225	2.0080	2.3880	1.5428	2.6329	-	-	-	-
		4	0.5053	2.8621	2.2035	1.6660	3.0700	-	-	-	-
	7	-	<b>2.1800</b>	<b>1.3300</b>	<b>2.7100</b>	<b>1.3600</b>	<b>2.6700</b>	<b>1.2700</b>	<b>1.7600</b>	-	-
		6	0.3768	3.5100	2.0600	2.0600	2.5300	1.4700	3.0600	-	-
		4	0.2207	5.7500	1.5700	2.3800	2.2200	1.5900	3.8100	-	-
		2	0.0842	14.980	0.7258	3.9600	1.4500	2.2100	5.4800	-	-
	9	-	<b>2.2000</b>	<b>1.3400</b>	<b>2.7400</b>	<b>1.3800</b>	<b>2.7600</b>	<b>1.3700</b>	<b>2.6900</b>	<b>1.2700</b>	<b>1.7700</b>
		6	0.1887	6.4000	1.6500	2.4900	1.8900	1.9000	2.6800	1.4300	3.7300
		4	0.0928	12.790	1.0600	3.1200	1.4600	2.3500	2.3900	1.6000	4.9300
		2	0.0550	21.540	0.7133	4.0800	1.1700	2.8700	2.0500	1.8200	5.9300

Tab. 1. The elements of Tschebyshev and Butterworth single terminated RLC filter prototype (Fig.1b), normalized to  $\omega=1$ , terminal resistor  $r=1 \Omega$

with serial resistors bringing effect of slope degradation in the stop-band. The computed characteristic are agree with ideal up to -80dB what is in practice fully acceptable in more cases.

Thus the active realization of serial lossy structures can be successfully used by synthesis with simple one op-amp synthetic elements with serial losses. This realizations can exhibit in some cases an advantage as better properties in the stop-band (smaller effect of transfer response degradation due to absence of parasitic transfer zeros).

	$r_s$	$h_1$	$c_1$	$l_2$	$c_2$	$l_3$
<b>B</b>	-	<b>1,5440</b>	<b>1,6936</b>	<b>1,3813</b>	<b>0,8940</b>	<b>0,3089</b>
	80m	1,2042	1,7040	1,4300	0,9200	0,3593
	100m	1,1423	1,7870	1,4611	0,9450	0,3258
<b>T</b>	-	<b>2,1500</b>	<b>1,3000</b>	<b>2,6200</b>	<b>1,2500</b>	<b>1,7400</b>
	50m	1,5300	1,3260	2,7780	1,3640	2,0740
	60m	1,2280	1,4210	2,9380	1,4250	2,1850

Tab. 2. The elements of Tschebyshev and Butterworth RLC ladder prototypes with serial lossy resistors

Using above prescribed optimization method have been calculated parameters of RLC ladder prototypes with serial losses in the case of Butterworth (B) and Tschebyshev (T) low – pass prototype of 5<sup>th</sup> order, which are summarized in table 2.

Starting from these parameters as illustrative example we have designed resulting filter topology with simple serial lossy FDNR active elements (Fig.6). An initial designed active filter structure has been optimized in final step of optimization procedure to compensate real parameters of active elements. How it is seen from resulting transfer response of designed low – pass filter (Fig.7), the resulting optimized magnitude response is identical with required (ideal) transfer response up value

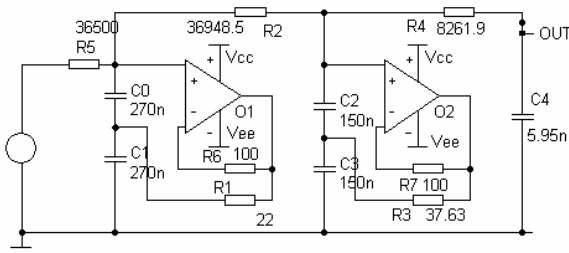


Fig.6. The resulting active low – pass Butterworth filter of 5<sup>th</sup> order with serial lossy FDNR elements about - 80dB, what is in practice fully acceptable result.

## 5. THE ELIMINATION OF REAL ACTIVE ELEMENT PROPERTIES

Most important advantage of goal - directed lossy frequency filter design is the possibility to

realize the active equivalent of the passive RLC ladder network with simple ( non – ideal) synthetic function block, like FDNR, GIC or synthetic

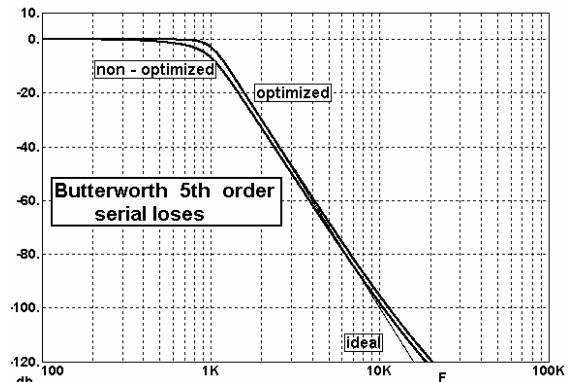


Fig. 7. The resulting transfer response in a case of network based on serial lossy resistor RLC prototype

inductors with minimized numbers of active elements [2]. The AC transfer response of filter designed from the RLC ladder prototype should exactly correspond with the required AC transfer response only if the ideal Op-amp or other active elements are used. The real frequency response of active elements caused shifting of the cut-of frequency and quality factor of filter. This fact is dominant by all active realizations (e.g. cascade realization etc.), if dominant real parameters of active elements are not taken in to account during the design procedure.

The above described usage of network optimizers can be used also to final correction of the

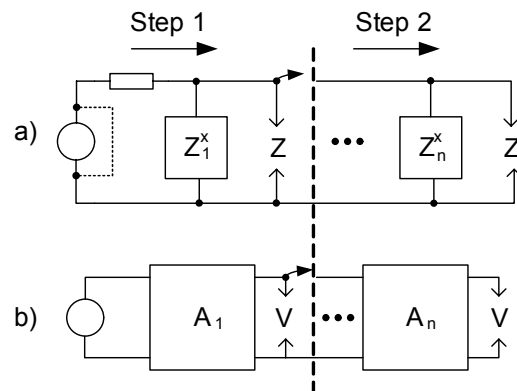


Fig.8. The principle of optimization method to compensation of real active elements properties by -a) non – cascade realization, b) cascade realization

active filter performance with real models of active elements. The process of the optimization must correspond with the principle of the used filter synthesis method.

By the case of non-cascade method of filter realization as the main optimized parameter during

synthesis optimization process was found cross-impedance of each sub-blocks (by the condition where the source generator is switched off). In the case of cascade filter realization was successfully used as optimized parameters AC voltage transfer function. The principle of active element optimization method and process of optimization procedure is roughly sketched in Fig. 8.

Applying above mentioned optimization method the non – cascade synthesized active filter from Fig.9, tab. 3 has been optimized to eliminate real properties of operational amplifiers. The low - pass RCD

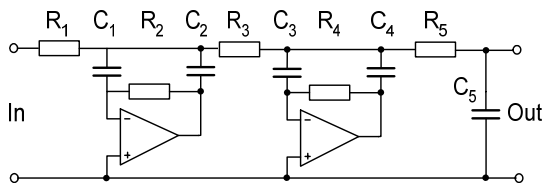


Fig. 9. The resulting active RCD structure of 5<sup>th</sup> order Tschebyshev low – pass filter

structure of the 5<sup>th</sup> filter order has been designed from the passive RLC lossy ladder prototype using Bruton's transformation [1]. The equivalent component values ( $R_1, C_1, \dots, R_5$ ) create initial vector of optimized parameters. The required AC transfer function was determined from ideal transfer response of network. From the Fig.10 we can observe the ideal characteristic (with ideal operational amplifier) and the characteristic obtained with real two-pole operational amplifier model LM741. From curves is very good seen the cut-off frequency shift due to the real properties of the operational amplifiers.

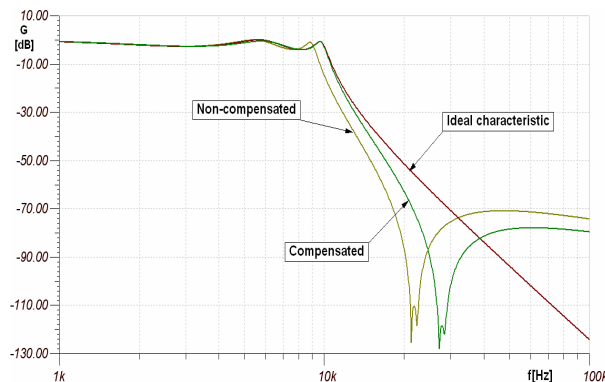


Fig.10. The transfer responses obtained by optimization process in case of active LP filter from Fig.7.

By the above presented compensation method (using Tina optimizer) was used to optimize the final network parameters as best goal function the cross – impedance of each block was found. Each active block was optimized separately, what increased the speed of optimization. The final optimized characteristic is shown in Fig.10.

During optimization process as most important part was identified procedure to determination of goal ideal AC transfer function (in tabled form) used by optimization procedure. This goal function must be specified in the transfer area, where are not presented the further parasitic effects of active elements. Therefore in the case of presented example of optimization process, the stop frequency of tabled AC transfer function has been selected the cut - off frequency of filter, it means 10kHz.

## CONCLUSION

In the paper there is described the simple network synthesis method which brings further possibility to improve filter design and optimization in area passive as well active filters. Here described method allows wider expansion of the filter synthesis based on goal – directed lossy filter prototypes. The presented method of compensation of active element influences can be widely used for all active elements like VFA, CFA, OTA, CCII etc. The great advantage of method is that enable to optimize resulting networks (including passive and active filters) by usage of usually accessible software for network analysis without requirement of special numerical programs what brings new possibility for many designers in area of network design and optimization.

## REFERENCES

- [1] Hájek K., Sedláček J. Kmitočtové filtry. Vydavatelství BEN Praha 2002
- [2] HÁJEK, K. - SEDLÁČEK, J.: Lossy LC Ladder Prototypes and their use for ARC Filter Optimization. WSEAS TRANSACTIONS on ELECTRONICS, Issue3, Volume2, July 2005, pp. 94-99, ISSN 1109- 9445.
- [3] MARTINEK, P.-DAŠA, T. Evolutionary algorithms by ARC filter synthesis. ECCTD 05, Cork, 2005, pp.155- 159.
- [4] PSPICE design software (<http://www.orcad.com/>)
- [5] PC Simulator Tina 6 pro ([www.tina.com](http://www.tina.com))

	$R_1$	$C_1$	$R_2$	$C_2$	$R_3$	$C_3$	$R_4$	$C_4$	$R_5$	$C_5$
Ideal	802,0	1,315n	241k	1,315n	4,224k	769p	440k	769p	4,256k	10n
Comp	833,54	490p	400,15k	1,791n	4,49k	458,1p	470,3k	1,109n	4,256k	9n

Tab 3 – ideal and optimized components value for 10 kHz low pass filter (fig 9, 10)